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OPTIMUM FLOWRATE HISTORY FOR COOLDOWN AND ENERGY STORAGE PROCESSES

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Abstract—This paper considers the basic thermal design problem of cooling or heating an object using the minimum amount of working fluid. It is shown analytically that if the duration of the cooling or heating process is fixed, then there exists a unique operating regime (flowrate–time function) which insures the minimum consumption of working fluid. The optimum flowrate is proportional to $(U/C_p)^{1/2}$, where U is the overall heat transfer coefficient and C_p is the specific heat of the working fluid; the optimum flowrate changes with time as $(U/C_p)^{1/2}$ varies with the temperature of the system. Numerical examples of cooldown and energy storage processes show that the implementation of this optimum operating regime can lead to significant savings in working fluid.

NOMENCLATURE

A, heat transfer area;
C, specific heat of object;
C_0, value of C at T_0;
C_p, specific heat of working fluid;
C^*, constant of integration;
F, integrand;
I_1, I_2, I_3, integrals;
m, total mass of coolant or hot fluid;
$\dot{m}$, flowrate;
M, mass of object;
NTU_0, number of heat transfer units based on U_0;
$\overline{NTU}$, number of heat transfer units based on $\bar{U}$;
p, exponent in specific heat function $C(T)$;
q, exponent in heat transfer coefficient function $U(T)$;
Q, heat transfer rate;
t, time;
t_c, cooldown time;
T, absolute temperature;
T_{out}, temperature of working fluid, inside the object;
T_H, high end-temperature of process;
T_L, low end-temperature of process;
T_0, initial temperature of working fluid;
U_0, heat transfer coefficient, $U(T_0)$;
$\bar{U}$, temperature-averaged heat transfer coefficient;
τ, absolute temperature ratio, T/T_0.

Subscripts

min, minimum;
opt, optimum;
o, c, optimum and constant flowrate regimes, respectively (used to characterize m and $\dot{m}$).

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INTRODUCTION

AN INCREASING number of novel installations for energy processing and conversion rely on the classical process of batch cooling or heating. For example, in the field of solar energy engineering, we find that the energy drawn from solar collectors can be temporarily stored by batch heating water tanks and underground porous rock beds [1]. Electric power companies are evaluating the possibility of storing thermal energy in large tanks of water or oil during slack periods, to ease the strain during heavy demand periods [2]. Furthermore, the emerging technology of large-scale superconducting devices relies heavily on the cooldown (batch cooling) of immense structures from room temperature (300 K) to the normal boiling point of helium (4.2 K); a noteworthy example in this group is the proposed construction of football stadium size superconducting structures for magnetic energy storage [3].

In all such applications, the expensive commodity is the fluid which must be pumped through the object (structure) of interest. The fluid heated in the boiler of a power plant inherits the price of the fuel which is burned. Likewise, the fluid heated in a solar collector is in limited supply, considering the high initial cost of installed solar collector area. And, finally, the cryogen (liquid helium) circulated through a superconducting structure is notoriously expensive, given the extreme ratio of absolute temperatures (300 K/4.2 K) which must be overcome by the liquefaction process [4].

In this paper we address the fundamental question of how to cool (or heat) a system to a desired temperature, by using the minimum quantity of precious fluid (cooling or heating agent). For clarity, we focus in detail on the cooldown process. The energy storage process is found to be analytically identical to the cooldown process and, for this reason, it is discussed only briefly at the end of the paper.

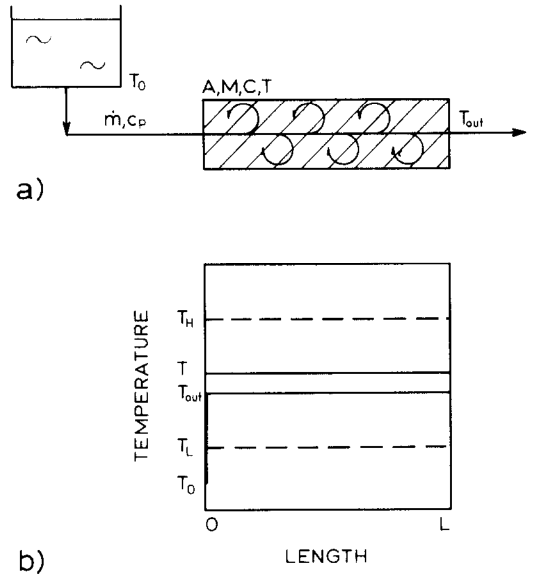

FIG. 1. Schematic of batch cooling system and temperature history during cooldown.

THE COOLDOWN PROCESS

The basic features of the model employed in our analysis are illustrated in Fig. 1. In the top drawing we show the coolant supply, which is at a temperature T_0. Next to it, we show the object (porous structure or pool of liquid) which must be cooled from an initial temperature T_H down to a final temperature T_L. Assuming that the coolant floods the object uniformly, we model the object's temperature as a function of time only, $T(t)$. This modeling feature is particularly good in the case of porous superconducting structures in which, in addition to the multidirectional channels available for the dispersion of coolant, the structure itself contains a substantial amount of high conductivity copper [3]. In the case of liquid pools for energy storage, the uniform T assumption means that the liquid is well-mixed (by free convection or other mechanisms).

In our model, the mass of the system is M and its specific heat is C. The total contact area available for object–coolant heat transfer is A. The instantaneous heat transfer rate is proportional to object–coolant temperature difference

$$Q = UA(T - T_{out}) \tag{1}$$

where, in general, the overall heat transfer coefficient U may be a function of temperature. Inside the object, the coolant is modeled as well-mixed, at a temperature which varies with time, $T_{out}(t)$. A schematic of the object and coolant temperature history is presented in the lower half of Fig. 1.

The link between the coolant supply T_0 and the object $T(t)$ is provided by the coolant stream of flowrate $\dot{m}$ and heat capacity C_p. The flowrate $\dot{m}$ can vary with time, such that at the end of the cooldown process the overall coolant requirement

$$m = \int_0^{t_c} \dot{m}(t)\,dt \tag{2}$$

is minimum. If the coolant flowrate is time-independent, $\dot{m}_c$, and if the heat transfer coefficient is independent of temperature, U_0, it is easy to show that

$$\frac{T(t) - T_0}{T_H - T_0} = \exp\left[-\frac{U_0 A}{MC}\frac{t}{1 + NTU_0}\right] \tag{3}$$

where $NTU_0 = U_0 A/(\dot{m}_c C_p)$. For the cooldown time t_c associated with the temperature drop $T_H \to T_L$ we obtain

$$t_c = MC\left(\frac{1}{U_0 A} + \frac{1}{\dot{m}_c C_p}\right) \ln\frac{T_H - T_0}{T_L - T_0} \tag{4}$$

hence, the overall coolant requirement is

$$m = \frac{MC}{C_p}\left(\frac{\dot{m}_c C_p}{U_0 A} + 1\right) \ln\frac{T_H - T_0}{T_L - T_0}. \tag{5}$$

Equation (5) shows that the coolant requirement m decreases steadily if the flowrate $\dot{m}_c$ decreases: in the limit $\dot{m}_c \to 0$, the coolant mass m reaches the asymptotic minimum value

$$m_{min} = \frac{MC}{C_p} \ln\frac{T_H - T_0}{T_L - T_0}. \tag{6}$$

Although desirable from a coolant-conservation perspective, the process which consumes only m_{min} is not practical because, as indicated in equation (4), it would require an infinitely long time. *Real* cooldown and energy storage processes face t_c = constant as a *constraint*. This constraint is obvious in the case of energy storage units for solar and peak-shaving applications, where t_c is measured between precise hours of the day.

THE OPTIMUM FLOWRATE SUBJECT TO THE FIXED-TIME CONSTRAINT

In the general case where the flowrate $\dot{m}$ is a function of time and the overall heat transfer coefficient U is a function of temperature, we can write

$$UA(T - T_{out}) = \dot{m}C_p(T_{out} - T_0). \tag{7}$$

Eliminating T_{out} between equation (7) and the first law of thermodynamics applied to the object,

$$MC\dot{T} = -UA(T - T_{out}), \tag{8}$$

yields an expression for the instantaneous flowrate

$$\dot{m} = \frac{M\dfrac{C}{C_p}\dot{T}}{T_0 - \dfrac{MC}{UA}\dot{T} - T}. \tag{9}$$

The total coolant mass requirement m is obtained by integrating equation (9) over the known cooldown interval t_c. This result may be written as a temperature integral between the corresponding temperature limits T_H and T_L,

$$m = \int_{T_H}^{T_L} \frac{M\dfrac{C}{C_p}\mathrm{d}T}{T_0 - \dfrac{MC}{UA}\dot{T} - T}. \qquad (10)$$

The optimum flowrate function $\dot{m}(t)$ which minimizes m is found indirectly, by first determining $T(t)$ appearing in equation (9). According to the calculus of variations [5], integral (10) is minimized if its integrand, named F, satisfies the following Euler equation for an extremal,

$$\frac{\mathrm{d}}{\mathrm{d}T}\left[\frac{\partial F}{\partial\left(\dfrac{\mathrm{d}t}{\mathrm{d}T}\right)}\right] - \frac{\partial F}{\partial t} = 0. \qquad (11)$$

Note that $F(\mathrm{d}t/\mathrm{d}T, T)$, where T is the independent variable and the optimum function $t(T)$ is to be determined. Solving equation (11) we obtain

$$\dot{T}_{\mathrm{opt}} = \frac{UA}{MC}\frac{T_0 - T}{1 + (C^*UA/C_p)^{1/2}} \qquad (12)$$

where the constant of integration C^* has the dimension s/kg. The value of constant C^* is determined by integrating equation (12) from $t = 0 (T = T_H)$ to $t = t_c(T = T_L)$, where U is a known function of temperature. Finally, inserting equation (12) into equation (9) we obtain

$$\dot{m}_{\mathrm{opt}} = (UA/C_pC^*)^{1/2}. \qquad (13)$$

This is a compact result of interesting physical significance. Bearing in mind that U varies as the average temperature (T, T_{out}) decreases, we learn that during relatively poor heat transfer conditions (low U) the mass flowrate should be decreased: this decrease is necessary in order to avoid the decrease in T_{out} and the corresponding drop in heat exchanger effectiveness. If during the same cooldown process the specific heat of coolant increases, then $\dot{m}$ must again decrease in order to avoid a further drop in heat exchanger effectiveness. Note also that $\dot{m}_{\mathrm{opt}}(t)$ is not directly a function of the object's temperature T. However, $\dot{m}_{\mathrm{opt}}$ depends on T through U and C_p.

COOLANT REQUIREMENTS FOR OPTIMUM VERSUS CONSTANT FLOWRATES

The savings in coolant mass m associated with employing the optimum flowrate history (13) instead of a constant rate $\dot{m}_c$ are calculated as the mass ratio m_o/m_c. In the following analysis, subscripts o and c refer to the optimum and constant flowrate regimes, respectively.

Combining equations (12) and (10) we obtain

$$m_o = \int_{T_H}^{T_L}\left[1 + \left(\frac{C_p}{C^*UA}\right)^{1/2}\right]\frac{MC\mathrm{d}T}{C_p(T_0 - T)}. \qquad (14)$$

The other mass requirement, m_c, is obtained by eliminating $\dot{T}$ between equations (9) and (10)

$$m_c = \int_{T_H}^{T_L}\left(1 + \frac{\dot{m}_cC_p}{UA}\right)\frac{MC\mathrm{d}T}{C_p(T_0 - T)}. \qquad (15)$$

The unknown constant C^* appearing in equation (14) is determined from the condition of equal cooldown times

$$\int_{T_H}^{T_L}\frac{\mathrm{d}T}{\dot{T}_{\mathrm{opt}}} = \frac{m_c}{\dot{m}_c} \qquad (16)$$

where the LHS corresponds to the optimum flowrate regime and the RHS to the constant flowrate regime. Substituting equations (12) and (15) for $\dot{T}_{\mathrm{opt}}$ and m_c in equation (16) leads to the integral condition

$$\int_{\tau_H}^{\tau_L}(C^*/UAC_p)^{1/2}\frac{MC\mathrm{d}\tau}{1-\tau} = \int_{\tau_H}^{\tau_L}\frac{MC\mathrm{d}\tau}{\dot{m}_cC_p(1-\tau)} \qquad (17)$$

where τ is the absolute temperature ratio T/T_0. The integrals appearing in equation (17) have been evaluated numerically, assuming that the object's heat capacity C and the heat transfer coefficient U vary with the absolute temperature as

$$C = C_0\tau^p \qquad (18)$$

and

$$U = U_0\tau^q. \qquad (19)$$

Finally, equation (17) yields

$$C^{*1/2} = \frac{I_1}{I_2}\frac{(U_0AC_p)^{1/2}}{\dot{m}_cC_p} \qquad (20)$$

with the notation

$$I_1 = \int_{\tau_L}^{\tau_H}\frac{\tau^p}{\tau - 1}\mathrm{d}\tau,\quad I_2 = \int_{\tau_L}^{\tau_H}\frac{\tau^{(p-q/2)}}{\tau - 1}\mathrm{d}\tau. \qquad (21, 22)$$

Based on this result, the coolant requirements (14, 15) can be expressed as

$$m_o = \frac{MC_0}{C_p}\left(I_1 + \frac{\dot{m}_cC_p}{U_0A}\frac{I_2^2}{I_1}\right) \qquad (23)$$

and

$$m_c = \frac{MC_0}{C_p}\left(I_1 + \frac{\dot{m}_cC_p}{U_0A}I_3\right) \qquad (24)$$

where I_3 is the integral

$$I_3 = \int_{\tau_L}^{\tau_H}\frac{\tau^{(p-q)}}{\tau - 1}\mathrm{d}\tau. \qquad (25)$$

The coolant mass ratio is obtained by dividing equation (23) and (24),

$$\frac{m_o}{m_c} = \frac{NTU_0 + (I_2/I_1)^2}{NTU_0 + I_3/I_1} \qquad (26)$$

where NTU_0 is the number of heat transfer units based on $\dot{m}_c$ and U_0,

$$NTU_0 = \frac{U_0 A}{\dot{m}_c C_p} \tag{27}$$

It is easy to verify that if U is constant (i.e., $q = 0$), the mass ratio equals unity, because integrals I_2 and I_3 reduce to I_1. In general, however, the exponents p and q are finite and m_o/m_c must be evaluated numerically. The results of this effort are presented in the next section.

RESULTS AND DISCUSSION

As an application of the optimum cooldown regime prescribed by equation (13), we considered the cooling of a large-scale superconducting structure. This process is characterized by the following parameters: $T_0 = 4.2$ K, the boiling point of helium; $T_L = 4.5$ K; $T_H = 80$ K, provided by liquid nitrogen precooling; $p = 2.85$, derived from heat capacity data of Al, Fe and Cu below 80 K, as a good approximation [6]. The relationship between U and T depends on the heat transfer mechanism, hence, it varies from one application to another. For this reason, we produced general information by varying exponent q from 0.1 to 10.

Figure 2 shows the coolant mass ratio for three discrete values of NTU_0. As q increases, the mass ratio m_o/m_c goes through a minimum located in the vicinity of $q = 1$. At first, the right side (rising) part of the curve appears inexplicable: increasing q means a stronger temperature dependence of U, hence, the savings associated with using $\dot{m}_{opt}$ should be enhanced. The reason for the rising part of the curve is that it is drawn for $NTU_0 =$ constant, where NTU_0 is based on $U_0 = U(T_0)$. Therefore, as q increases and as U_0 is held fixed, the effective (average) U available for object–coolant heat transfer during cooldown increases significantly.

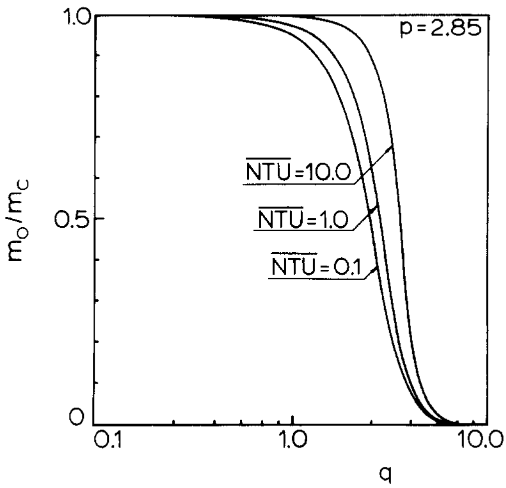

FIG. 3. Coolant mass ratio m_o/m_c vs heat transfer coefficient exponent q, for different values of the mean number of heat transfer units $\overline{NTU}$.

In order to evaluate the $(m_o/m_c) - q$ dependence for a cooldown process where the effective thermal contact is fixed, we defined the mean heat transfer coefficient

$$\bar{U} = \frac{1}{\tau_H - \tau_L} \int_{\tau_L}^{\tau_H} U(\tau)\,d\tau \tag{28}$$

and held $\overline{NTU} = \bar{U}A/(\dot{m}_c C_p)$ constant as we varied q. The results are shown in Fig. 3. The mass ratio m_o/m_c drops off dramatically above a certain, critical, value of exponent q. From an engineering standpoint, we see stronger incentives for using the optimum flowrate history (13) in cases in which q is large and/or the average $\overline{NTU}$ is small. This conclusion is in agreement with the qualitative discussion which followed equation (13).

The effect of heat capacity exponent p on the mass ratio is illustrated in Fig. 4. This graph corresponds to one value of $\overline{NTU}$, and shows the same dramatic drop

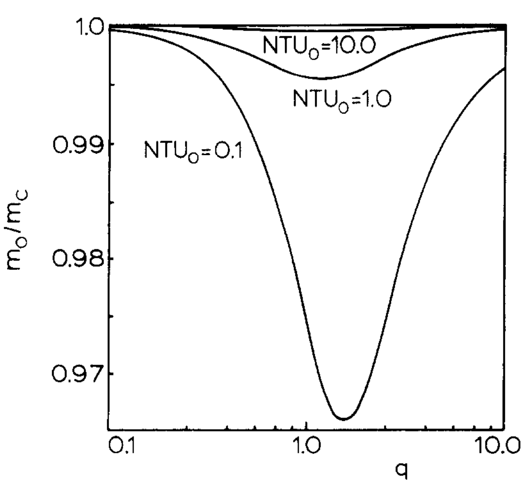

FIG. 2. Coolant mass ratio m_o/m_c vs heat transfer coefficient exponent q, for different values of NTU_0.

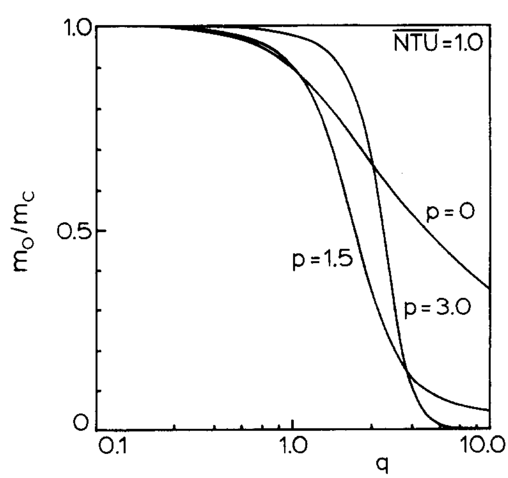

FIG. 4. Coolant mass ratio m_o/m_c vs heat transfer coefficient exponent q, showing the effect of specific heat exponent p.

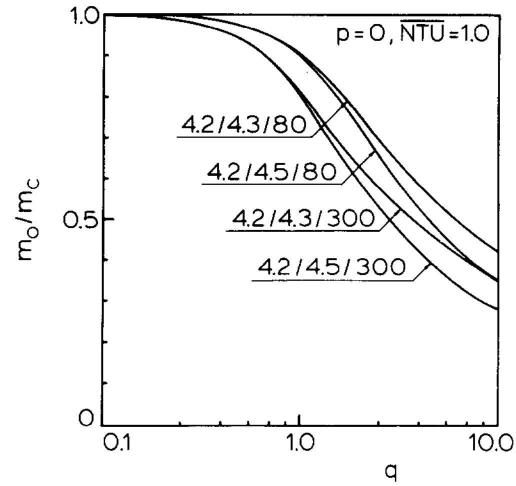

FIG. 5. Coolant mass ratio m_o/m_c vs heat transfer coefficient exponent q, showing the effect of end temperatures T_0, T_L, T_H (numbers on the figure are in corresponding order).

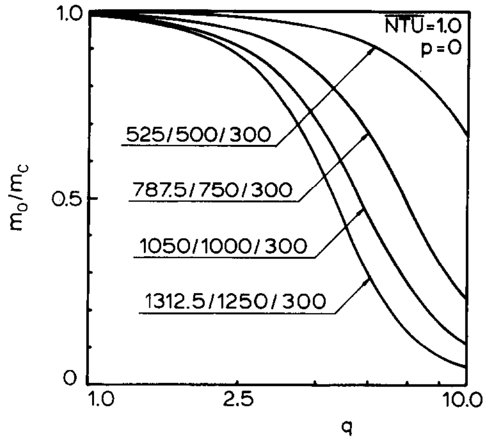

FIG. 7. Hot fluid mass ratio m_o/m_c vs heat transfer coefficient exponent q, showing the effect of end temperatures T_0, T_H, T_L (numbers on the figure are in corresponding order).

in m_o/m_c as q increases. The abruptness of the drop is enhanced as exponent p increases.

The effect of varying τ_H and τ_L is presented in Fig. 5, for the special case $p = 0$ and $\overline{NTU} = 1$. The mass ratio drops steadily as exponent q increases. Changes in both τ_H and τ_L lead to measurable changes in the coolant savings associated with using $\dot{m}_{opt}$. For example, the largest savings are recorded as both τ_L and τ_H increase. In connection with the economic cooldown of a large superconducting structure, the optimum flowrate is recommended especially when 4.2 K helium gas is used throughout the cooldown process (from T_H = 300 K to T_L), without liquid nitrogen precooling.

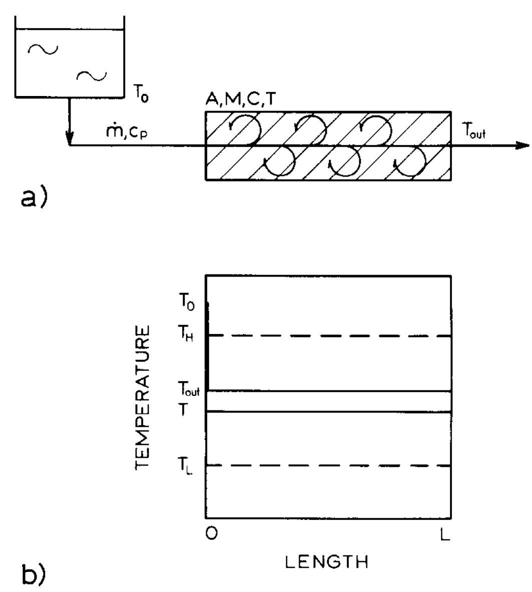

FIG. 6. Schematic of batch heating system and temperature history during energy storage process.

THE HEATING PROCESS

The minimization of overall heating fluid consumption can be pursued in the same manner in which we analyzed the cooldown process. Figure 6 shows the basic model which applies to the heating process. Hot fluid of temperature T_0 heats the object from the initial (low) temperature T_L to the final (high) temperature T_H. It can be shown that the equations describing the relationship between flowrate history, total hot fluid requirement and temperature history are identical to the equations developed for the cooldown process. The only difference occurs in the temperature limits of integration.

In the interest of brevity, we omit the analysis and show only a set of representative results. Figure 7 reports the changes in the mass ratio m_o/m_c as the heat transfer coefficient q increases ($\overline{NTU}$ and p are held constant). The behavior of m_o/m_c is qualitatively the same as in Fig. 5 for the cooldown process. Comparing Figs. 7 and 5, we learn that for a given q the heating fluid mass ratio (Fig. 7) is greater than the cooling mass ratio (Fig. 5). This means that the optimum flowrate history (13) is less effective as a fluid-saving method during heating processes, relative to its application to cooldown processes. Numerically, the difference is due to the fact that heating processes occur above room temperature, therefore, they involve smaller absolute temperature ratios (τ) than the cryogenic cooldown example illustrated in the preceding section.

CONCLUDING REMARKS

In this paper we considered the basic thermal design question of cooling/heating a thermal mass while using the minimum amount of working fluid. We showed that when the time interval alloted to the process is fixed, there exists a unique regime of operation which insures the largest savings in working fluid. We discussed quantitatively the nature of this optimum regime, by focusing on the flowrate history for a cooldown process. The optimum flowrate history is influenced by the temperature dependence of the object–coolant heat transfer coefficient and by the specific heat of the working fluid.

The payoff associated with using the optimum flowrate, equation (13), was evaluated as the mass ratio m_0/m_c. By means of specific cooldown and heating examples, we showed that there exist definite parametric domains (applications) in which the savings in working fluid are substantial. In addition to specific examples, this paper provides the heat transfer engineer with general analytical results which are sufficient for determining the optimum cooling/heating regime for any application.

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DEBIT OPTIMAL POUR LES MECANISMES DE REFROIDISSEMENT ET DE STOCKAGE D'ENERGIE

Résumé—On considère le problème fondamental du refroidissement ou de chauffage d'un objet en utilisant la quantité minimale de fluide de travail. On montre analytiquement que si la durée du refroidissement ou du chauffage est fixée, il existe alors un régime opératoire unique (fonction débit-temps) qui assure la consommation minimale de fluide. Le débit optimal est proportionnel à $(U/C_p)^{1/2}$ où U est le coefficient global de transfert et C_p la chaleur massique du fluide; le débit optimal change avec le temps quand $(U/C_p)^{1/2}$ varie avec la température du système. Des exemples numériques de refroidissement et de stockage d'énergie montrent que l'implantation de ce régime opératoire optimal peut conduire à des économies sensibles de fluide de travail.

OPTIMALE DURCHFLUSSCHARAKTERISTIK FÜR ABKÜHL- UND HEIZVORGÄNGE

Zusammenfassung—Ein Grundproblem der Wärmetechnik stellt sich mit der Frage, wie ein Objekt abgekühlt oder erwärmt werden muß, so daß ein Minimum an Arbeitsfluid verbraucht wird. In diesem Beitrag wird analytisch gezeigt, daß bei einer vorgegebenen Dauer des Kühl- oder Heizprozesses eine bestimmte Prozessführung (Durchfluß-Zeit-Charakteristik) existiert, die einen minimalen Arbeitsfluidverbrauch gewährleistet. Der optimale Fluiddurchsatz verhält sich proportional zu $(U/C_p)^{1/2}$, wobei U den mittleren Wärmeübergangskoeffizienten und C_p die spezifische Wärmekapazität des Arbeitsmittels bezeichnet. Der optimale Durchfluß ist zeitabhängig, indem sich $(U/C_p)^{1/2}$ mit der Temperatur bei der Abkühlung/Heizung ändert. Zahlenbeispiele für Abkühl- und Heizprozesse legen dar, daß die Anwendung dieser Vorschrift für die Durchflußsteuerung zu beträchtlichen Einsparungen an Arbeitsmittel führen kann.

ИЗМЕРЕНИЕ ОПТИМАЛЬНОЙ СКОРОСТИ ТЕЧЕНИЯ ПРИ ОХЛАЖДЕНИИ И НАГРЕВАНИИ

Аннотация — Рассматривается важная проблема теплового расчета процесса охлаждения или нагревания объекта минимальным количеством рабочей жидкости. Аналитически показано, что для заданной длительности процесса охлаждения или нагревания существует единственный рабочий режим (функция скорость течения — время), при котором расход рабочей жидкости минимален. Оптимальная скорость течения пропорциональна $U/C_p{}^{1/2}$, где U — суммарный коэффициент переноса тепла, а C_p — удельная теплоемкость рабочей жидкости; оптимальная скорость течения изменяется со временем по мере того, как величина $U/C_p{}^{1/2}$ изменяется с изменением температуры системы. Численные примеры процессов отбора и подвода тепла показывают, что использование оптимального режима ведет к значительной экономии рабочей жидкости.